Supplementary Materials for "The Economics of Web Search"

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A Job Search Duration and Wage Dispersion

Our paper has explored a new two factor undiscounted search model. We now show that dispersion still predicts search duration with discounting, albeit with a subtle change. We consider McCall's classic discounted job search model. The big picture is that discounting adds another opportunity cost of search, and greater wage dispersion can lower or raise this cost, and so job search duration can rise or fall.

Call F_B more *log-dispersed* than F_A if $\log(F_B^{-1})$ is weakly steeper than $\log(F_A^{-1})$.[41] These orders typically agree: Table 2 gives examples of several parametric distributions easily ordered by both dispersion and log-dispersion.

Consider a generalized job search model with discount factor $\beta \leq 1$ and search cost $c > 0$. Assume wages have density $f = F'$ on $(0, b)$, for $b \leq \infty$. The reservation wage $\bar{w}(c)$ obeys

$$(1-\beta)\bar{w}(c) = -c + \beta \int_{\bar{w}(c)}^{\infty} [1 - F(w)]dw. \tag{62}$$

We now enrich the comparative static of Theorem 3 for payoff discounting.

Proposition 1 *Suppose the wage distribution on $[0, \infty)$ changes from F_A to F_B, which is more dispersed. Let $\mathcal{S}_i(c) \equiv 1 - F_i(\bar{w}_i(c))$ be the stopping chance for $i = A, B$.*

(a) If $\beta = 1$, search duration rises in dispersion. If $\beta < 1$, it rises at high search costs, and falls at low search costs: $\mathcal{S}_B(c) < \mathcal{S}_A(c)$ iff $c > \bar{c}$ for some threshold $\bar{c}$.

[41] If F_B is more log-dispersed than F_A and has a higher lower support, then F_B is more dispersed. For if $F_B^{-1}(a) \geq F_A^{-1}(a)$ at some a, then $\partial F_B^{-1}(a)/\partial a > \partial F_A^{-1}(a)/\partial a$ by log-dispersion. So if $F_B^{-1}(0) \geq F_A^{-1}(0)$, then $F_B^{-1}(a) \geq F_A^{-1}(a)$ at all $a \in (0, 1)$, and $\partial F_B^{-1}(a)/\partial a > \partial F_A^{-1}(a)/\partial a$ at all $a \in (0, 1)$.

Distribution	cdf	Support	Dispersed and log-dispersed if
Exponential	$1-e^{\lambda z}$	$[0,\infty)$	$\lambda\uparrow$
Gamma	$\frac{1}{\Gamma(k)}\gamma(k,z/\theta)$	$[0,\infty)$	$\theta\uparrow$
Log-normal	$\frac{1}{2}+\frac{1}{2}\text{erf}\left(\frac{\log(z)-\mu}{\sigma\sqrt{2}}\right)$	$[0,\infty)$	$\mu\uparrow$
Type-2 Gumbel	$e^{-bz^{-a}}$	$[0,\infty)$	$(i)a\downarrow$ and $z>e^{-b}$ or $(ii)b\uparrow$
Pareto distribution	$1-(1+\lambda z)^{1/\lambda}$	$[0,\infty)$	$\lambda\uparrow$
Uniform	$(z-a)/(b-a)$	$[a,b]$	$(i)a\downarrow$ or $(ii)b\uparrow$

Table 2: **Dispersion and Log-dispersion of Probability Distributions.**

(*b*) *Search duration rises in log-dispersion.*

Proposition 1 implies Theorem 3 with no discounting: With discounting, search duration rises in dispersion for high search costs, and rises for all search costs given log-dispersion. We depict Proposition 1 with numerical examples in Figure 12.[42]

Example. Assume net wages $W=\xi Y-\Delta$, where wage $Y>0$ has cdf G, $\xi>0$ scales Y, and $\Delta>0$ is the disutility of work. Changing variables $w=\xi y-\Delta$ in (62) yields

$$(1-\beta)\bar{y}=-\left[\frac{c-\Delta(1-\beta)}{\xi}\right]+\beta\int_{\bar{y}}^{\infty}[1-G(y)]dy. \tag{63}$$

Notice how greater wage dispersion ξ has a similar effect as lower search disutility Δ — i.e., it lowers the (bracketed) search cost and spurs search, reducing search duration.

This logic holds if $c\geq\Delta(1-\beta)$ — so that search is costly. Once $c<\Delta(1-\beta)$, the worker profits from search, and these profits fall in ξ, reducing search duration.

In this example, *log-dispersion falls in* ξ. For if $a=G(y)$, and $w=\xi G^{-1}(a)-\Delta>0$:

$$\frac{\partial\log[F^{-1}(a)]}{\partial a}=\frac{\partial\log[\xi G^{-1}(a)-\Delta]}{\partial a}=\frac{1}{g[G^{-1}(a)][G^{-1}(a)-\Delta/\xi]}.$$

As ξ rises, both $g[G^{-1}(a)]$ and $G^{-1}(a)$ are fixed, and the right side falls. Thus, *log-dispersion falls.* In this case, Proposition 1 (*a*) holds, but part (*b*) does not.

Consider how convex transformations of a r.v. impact dispersion and log-dispersion:

Lemma 6 *If ϕ is an increasing convex function with $\phi(0)\geq 0$ and $\phi'(0)\geq 1$, then $Y\equiv\phi(X)$ is more dispersed and log-dispersed than random variable $X>0$.*

[42] In the right panel, $W_1\sim U[0,1]$ and $W_2=\phi(W_1,1.5)$ where $\phi(x,t)=x+x^t/t$. In the left panel, $W_1=\phi(U,2.5)$ and $W_2=\phi(U,0.3)$, where U is uniform $[0,1]$. In the left panel, W_2 is more dispersed and less log-dispersed than W_1. In both panels, the discount factor is $\beta=0.9$.

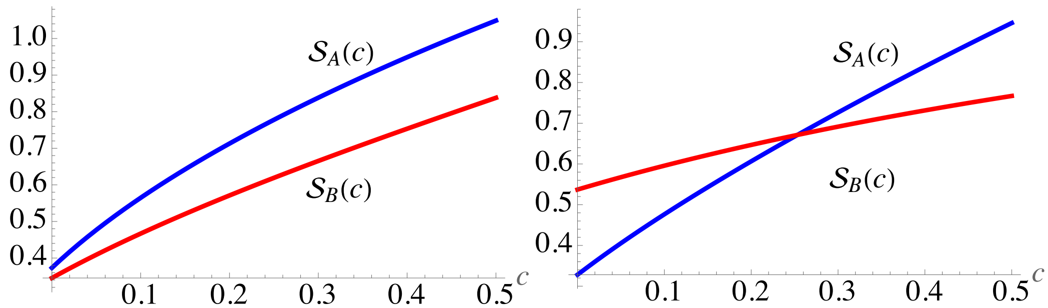

Figure 12: **Dispersion and Duration in the Job Search Model (Proposition 1).** We plot stopping chances $\mathcal{S}_A$ and $\mathcal{S}_B$ as a function of search cost c, with prize distribution F_B more disperse than F_A. At left, the wage distribution F_B is also more log-dispersed than F_A, and has a stopping chance at all $c > 0$. At right, F_B is not more log-dispersed than F_A, and stopping chance is higher for the more dispersed at low search costs c.

So scaling wages lifts search duration since $\phi(x) = \xi x$ obeys Lemma 6 if $\xi > 1$.

Proof of Lemma 6. The quantile function of Y is $\phi(F^{-1}(a))$, with slope $\phi'(F^{-1}(a))$ times $\partial F^{-1}(a)/\partial a$. If $\phi' > 1$, it is steeper than $F^{-1}(a)$, and Y is more dispersed.

Next, by $\phi'(0) > 0$ and the convexity of ϕ, we have $\phi'(x)/\phi(x) \geq 1/x$ at all $x > 0$. That Y is more log-dispersed than X follows from:

$$\frac{\partial}{\partial a}\log[\phi(F^{-1}(a))] = \frac{\phi'(F^{-1}(a))}{\phi(F^{-1}(a))}\frac{\partial F^{-1}(a)}{\partial a} \geq \frac{1}{F^{-1}(a)}\frac{\partial F^{-1}(a)}{\partial a} = \frac{\partial}{\partial a}\log[F^{-1}(a)] \qquad \square$$

Proof of Proposition 1 (a): From (62), $\bar{w}_i'(c) = -[1-\beta+\beta(1-F_i(\bar{w}_i(c)))]^{-1}$. Then

$$\frac{\partial \mathcal{S}_i(c)}{\partial c} = \frac{\partial[1-F_i(\bar{w}(c))]}{\partial c} = \frac{f_i(\bar{w}(c))}{1-\beta+\beta[1-F_i(\bar{w}_i(c))]}.$$

by the chain rule. If $c = \mathcal{C}_i(s)$ is the inverse function of $s = \mathcal{S}_i(c)$, then

$$\frac{\partial \mathcal{C}_i(s)}{\partial s} = \left(\frac{\partial \mathcal{S}_i(c)}{\partial c}\right)^{-1} = \frac{1-\beta+\beta\mathcal{S}_i(c)}{f_i[F_i^{-1}(1-\mathcal{S}_i(c))]}, \qquad (64)$$

Since $\partial F^{-1}(a)/\partial a = 1/f[F^{-1}(a)]$ rises in the dispersion of F, so does $\partial \mathcal{C}(s)/\partial s$. So $\mathcal{S}_B(c) - \mathcal{S}_A(c)$ is downcrossing: it has at most one sign change from + to − as c rises.

Assume undiscounted search. As $c \to 0$, Sam never stops searching: $\mathcal{S} \to 0$, or $\mathcal{S}_A$ and $\mathcal{S}_B$ cross at 0. As $\mathcal{S}_A(c)$ and $\mathcal{S}_B(c)$ downcross, $\mathcal{S}_B(c) < \mathcal{S}_A(c)$ for all $c > 0$.

Proof of Proposition 1 (*b*): As in (11), we can rewrite (62) as $\Gamma(F_i, \mathcal{S}_i(c)) = 0$ where

$$\Gamma(F_i, \mathcal{S}_i(c)) \equiv -(1-\beta)F_i^{-1}(1-\mathcal{S}_i(c)) - c + \beta \int_{1-\mathcal{S}_i(c)}^{1} (1-\alpha)\frac{\partial F_i^{-1}(\alpha)}{\partial \alpha} d\alpha. \quad (65)$$

A unique zero $\mathcal{S}_i(c)$ exists, since the right side rises in $\mathcal{S}_i(c)$. We claim $\mathcal{S}_A(c) < \mathcal{S}_B(c)$.

For simplicity, write $s_A = \mathcal{S}_A(c) = 1 - F_A(\bar{w}_A(c))$ and $s_B = \mathcal{S}_B(c) = 1 - F_B(\bar{w}_B(c))$.

Case 1: Assume $F_B^{-1}(1-s_A) \leq F_A^{-1}(1-s_A)$. Change F_A to F_B at $\mathcal{S}_i(c) = s_A$. By this case's premise, the first term on the right side of (65) rises. Since F_B^{-1} is steeper than F_A^{-1}, the right side integral in (65) rises shifting from F_A to F_B at $\mathcal{S}_i(c) = s_A$. Then $\Gamma(F_B, s_A) \geq \Gamma(F_A, s_A) = 0 = \Gamma(F_B, s_B)$. As $\Gamma(F_B, s)$ rises in s, we have $s_B \leq s_A$.

Case 2: Assume $F_B^{-1}(1-s_A) > F_A^{-1}(1-s_A)$. First, rewrite (65) as:

$$\frac{\Gamma(F_i, s_i)}{F_i^{-1}(1-s_i)} = -(1-\beta) - \frac{c}{F_i^{-1}(1-s_i)} + \beta \int_{1-s_i}^{1} (1-\alpha) \frac{\partial F_i^{-1}(\alpha)/\partial \alpha}{F_i^{-1}(\alpha)} \frac{F_i^{-1}(\alpha)}{F_i^{-1}(1-s_i)} d\alpha$$

Change F_A to F_B. By the premise of this case, the second term on the RHS rises. First, $[\partial F_i^{-1}(\alpha)/\partial \alpha]/F_i^{-1}(\alpha)$ in the integral also increases, because $\log(F_B^{-1})$ is steeper than $\log(F_A^{-1})$. Similarly, $F_i^{-1}(\alpha)/F_i^{-1}(1-s_i)$ rises at each $s_i = s_A$:

$$\log\left[\frac{F_i^{-1}(\alpha)}{F_i^{-1}(1-s_i)}\right] = \log[F_i^{-1}(\alpha)] - \log[F_i^{-1}(1-s_i)] = \int_{1-s_i}^{\alpha} \frac{\partial \log[F_i^{-1}(x)]}{\partial x} dx$$

The integrand rises pointwise as $\log(F_B^{-1})$ is steeper than $\log(F_A^{-1})$. Finally, as in Case 1, from $\Gamma(F_B, s_A) \geq 0$, we conclude $s_B \leq s_A$. □

B Mean-Preserving Spread: Search Can End Faster

Assume $-\mathcal{Z}$ is Pareto with shape parameter $\gamma > 1$ and $\mathcal{Z}$ has support $(-\infty, \bar{z}]$ with $\bar{z} < 0$. By the definition of the Pareto distribution the cdf of $\mathcal{Z}$ is $H(z) = (\bar{z}/z)^\gamma$ and its mean is given by $E[\mathcal{Z}] = \gamma\bar{z}/(\gamma-1)$. We restrict $E[\mathcal{Z}] = -1$ by setting $\bar{z} = -1 + 1/\gamma$. Then the cdf becomes $H(z) = [(-1+1/\gamma)/z]^\gamma$ and its support is $(-\infty, -1+1/\gamma]$. The density h and cdf H are both log-convex in this example.

Near $\gamma = 1$, if γ falls, then Claim 23 implies that $\mathcal{Z}$ incurs a MPS, while Claim 24 asserts that the stopping chance rises (i.e. search duration falls).

Claim 23 *If γ falls, then $\mathcal{Z}$ has a MPS, but $\mathcal{Z}$ does not grow more disperse if $\gamma < 2$.*

PROOF: Let $\gamma_B > \gamma_A > 1$. For $b \in (0,1)$ and $a = A, B$, the quantile function is $H_a^{-1}(b) = -b^{-1/\gamma_a}(1-\gamma_a)/\gamma_a$. Since the means of H_A and H_B are -1 by construction, if $H_A^{-1}(b) - H_B^{-1}(b)$ is single-crossing, then H_A is a MPS of H_B, by **?**. Since $H_A^{-1}(b) - H_B^{-1}(b) = H_A^{-1}(b)\left[1 - H_B^{-1}(b)/H_A^{-1}(b)\right]$, and $H_A^{-1}(b) < 0$, it suffices to show

$$\frac{H_B^{-1}(b)}{H_A^{-1}(b)} = \frac{\gamma_A(\gamma_B-1)}{\gamma_B(\gamma_A-1)} b^{\frac{1}{\gamma_A}-\frac{1}{\gamma_B}}$$

rises in b — which holds, as $\gamma_B > \gamma_A > 1$. So H_A is a mean-preserving spread of H_B.

Next consider the change in the slope of the quantile function with respect to γ

$$\frac{d}{d\gamma}\left[\frac{dH^{-1}(b)}{db}\right] = \frac{d}{d\gamma}\left[\frac{\gamma-1}{\gamma^2 b^{1/\gamma+1}}\right] = \frac{\gamma-1}{\gamma^2 b^{1/\gamma+1}}\left[\frac{2-\gamma}{\gamma(\gamma-1)} + \frac{1}{\gamma^2}\log(b)\right] \tag{66}$$

Since $\log(b) \in (-\infty, 0)$, for $\gamma \in (1,2)$, expression (66) is positive iff b is large enough. So $H^{-1}(b)$ does not flatten for all $b \in (0,1)$ as γ rises: Dispersion needn't rise in γ. □

Claim 24 (Rising γ) *If $c > 0$, then $1 - H(\zeta(c))$ falls in γ iff $\gamma < \gamma^*$, for $\gamma^* > 1$.*

PROOF: If $H(z) = [(-1+1/\gamma)/z]^\gamma$, the Bellman equation (3) becomes

$$(c+1)\gamma H(\zeta)^{1/\gamma} - \gamma + 1 - H(\zeta) = 0. \tag{67}$$

A unique solution, say $H_\gamma(\zeta)$ exists, as the LHS rises in $H(\zeta)$, is negative if $H(\zeta) = 0$, and positive if $H(\zeta) = 1$. Differentiating (67) in γ, we have $dH_\gamma(\zeta)/d\gamma \geq 0$ iff

$$H_\gamma(\zeta)^{1/\gamma}[1 - \log(H_\gamma(\zeta)^{1/\gamma})] \le \frac{1}{c+1}. \tag{68}$$

We claim inequality (68) is strict iff $\gamma < \gamma^*$, for $\gamma^* > 1$. Now, (68) is strict for γ near 1, since $x[1 - \log(x)] \downarrow 0$ as $x \downarrow 0$, and $H(\zeta) \downarrow 0$ as $\gamma \downarrow 1$ by (67). Once $\gamma > 1$, if (68) binds at some γ^*, then (68) is strict and $dH_\gamma(\zeta)/d\gamma > 0$ for $\gamma < \gamma^*$, and $dH_\gamma(\zeta)/d\gamma = 0$ at $\gamma = \gamma^*$. But then $H_\gamma(\zeta)^{1/\gamma}$ rises in γ at $\gamma = \gamma^*$, and so (68) rises in γ at $\gamma = \gamma^*$. So, (68) fails for all $\gamma > \gamma^*$, proving that $dH_\gamma(\zeta)/d\gamma > 0$ iff $\gamma < \gamma^*$. □

C Necessity of the Dispersive Order

The dispersive order is sufficient but not necessary for search duration to increase. Indeed, as shown in the left side of (11), the stopping chance depends on the slope of the quantile function at all quantiles above $1-S(c)$. Hence search duration might increase even if the quantile function becomes flatter in some regions. Now we explain under what conditions will the dispersive order become necessary. When making decision from experience, individuals sometimes underweight the probability of rare events, see Barron and Erev (2003) and Hertwig et al. (2004) for experimental evidence. Consider a searcher who ignores the extreme realizations of $\mathcal{Z}$. Specifically, for some $0 \leq \beta < \alpha \leq 1$, Sam thinks all offers above quantile α or below quantile β are impossible, i.e. the search thinks the distribution is $[H(z)-b]/(a-b)$ for $z \in [H^{-1}(\beta), H^{-1}(\alpha)]$. When drawing any $z < H^{-1}(\beta)$ or $z > H^{-1}(\alpha)$, Sam thinks it is a measure zero event. A searcher that neglects rare events is characterized by a pair of quantiles and a search cost, i.e. (α, β, c).

Theorem 11 *Search duration rises for all searchers that neglect rare events if and only if prize dispersion rises.*

PROOF: If the hidden factor distribution changes from H_1 to H_2, we argue that search duration rises for any (α, β, c) iff H_2 is more dispersed than H_1.

Dispersion $\implies$ Longer duration: For $j = 1, 2$ let $\hat{H}_j$ be the distribution of offers that Sam perceives. The corresponding quantile function satisfies $\hat{H}_j^{-1}(a) = H_j^{-1}[a(\alpha-\beta)+\beta]$. Hence $\hat{H}_2$ is more dispersive than $\hat{H}_1$ when H_2 is more dispersive than H_1. By Theorem 3, search duration is higher under H_2.

Longer duration $\implies$ Dispersion: Suppose H_2^{-1} is flatter than H_1^{-1} for some open interval (d_1, d_2). Consider a searcher who has $\beta = 0$ and $\alpha = d_2$. Then (11) becomes

$$c = \int_{\hat{H}_i(\hat{\zeta}_i)}^{1} (1-a) \frac{\partial \hat{H}_i^{-1}(a)}{\partial a} da = \int_{\hat{H}_i(\hat{\zeta}_i)(\alpha-\beta)+\beta}^{d_2} \left(1 - \frac{s-\beta}{\alpha-\beta}\right) \frac{\partial H_i^{-1}(s)}{\partial \alpha} ds$$

where the right side changes variable $s = a(\alpha-\beta)+\beta$. For sufficiently small c, Sam's continuation chance $\hat{H}_i(\hat{\zeta}_i)(\alpha-\beta)+\beta \in (d_1, d_2)$ for $i = 1, 2$. Since $H_2^{-1}(s)$ is flatter than $H_1^{-1}(s)$ for $s \in (d_1, d_2)$, $\hat{H}_2(\hat{\zeta}_2) < \hat{H}_1(\hat{\zeta}_1)$ and so search duration is lower under H_2. □

D Quitting and Dispersion

More dispersion of the hidden factor accelerates quitting iff the quit payoff is low:

Theorem 12 (Quitting Chance) *If the hidden factor $\mathcal{Z}$ dispersion rises, then q rises iff $u < \bar{u}$, some $\bar{u} \in \mathbb{R} \cup \pm\infty$. If it is a mean preserving dispersion, then $|\bar{u}| < \infty$.*

For a quick intuition, assume just one inside option $(\mathcal{X}, \mathcal{Z})$. Sam quits if $\mathcal{X} + \min(\mathcal{Z}, \zeta(c)) \leq u$; so he doesn't participate $(\mathcal{X} + \zeta(c) \leq u)$ or declines the inside option $(\mathcal{X} + \mathcal{Z} < u)$. For a mean preserving dispersion of $\mathcal{Z}$, the lower $\mathcal{Z}$ quantiles fall and $\zeta(c)$ rises. So $P(\mathcal{X} + \min(\mathcal{Z}, \zeta(c)) \leq u)$ rises for small u and falls for large u.

After a mean preserving dispersion, the quitting chance rises for low fallbacks, and otherwise falls (Theorem 12). This speaks to classic search: For product search — where one buys for all prices, as $\bar{u} \ll 0$ — dispersion leads one to quit. For job search — the second case, as one might not take a job — dispersion deters quitting.

Proof of Theorem 12: Let $\mathcal{Z}_B$ be a mean preserving dispersion of $\mathcal{Z}_A$, with respective cdfs H_B and H_A. The quitting chance is $q_a = \pi_a(u - \zeta(c), c)^N$, for $a = A, B$ by (43). It suffices that $q_B = \pi_B(u - \zeta_B(c), c) \gtreqless \pi_A(u - \zeta_A(c), c) = q_A$ as $u \lesseqgtr \bar{u}$, some $\bar{u}$.

Let $\underline{H}_a$ be the cdf of $\min\{\mathcal{Z}_a, \zeta_a(c)\}$, for $a = A, B$. Posit $\zeta_B(c) \geq \zeta_A(c)$. If $z < \zeta_A(c)$, then $\underline{H}_B(z) - \underline{H}_A(z) = H_B(z) - H_A(z)$. This is *downcrossing* (crosses at most once from + to −), since H_B^{-1} is steeper than H_A^{-1}. Likewise, $\underline{H}_B(z) - \underline{H}_A(z) = H_B(z) - 1 \leq 0$ for $z \in [\zeta_A(c), \zeta_B(c))$, and $\underline{H}_B(z) - \underline{H}_A(z) = 0$ for $z > \zeta_B(c)$. So $\underline{H}_B - \underline{H}_A$ is downcrossing in this case. Lastly, we similarly deduce that $\underline{H}_B - \underline{H}_A$ is downcrossing when $\zeta_B(c) \leq \zeta_A(c)$.

We can rewrite $\pi(u - \zeta(c), c) \equiv P(\min(\mathcal{Z}, \zeta(c)) \leq u - \mathcal{X})$ as:

$$\pi(u - \zeta(c), c) = \int_{-\infty}^{\infty} P(\{\min(\mathcal{Z}, \zeta(c)) \leq s\} \cap \{s = u - \mathcal{X}\}) ds = \int_{-\infty}^{\infty} \underline{H}_a(s) g(u - s) ds.$$

Since $\underline{H}_B(s) - \underline{H}_A(s)$ is downcrossing, so is $\pi_B(u - \zeta_B(c), c) - \pi_A(u - \zeta_A(c), c) = \int_{-\infty}^{\infty} [\underline{H}_B(s) - \underline{H}_A(s)] g(u - s) ds$, as g is a log-concave pdf (Karlin and Rubin, 1955).

Integrating (3) by parts, $\zeta_a(c) = -c + E[\mathcal{Z}] + \int_{-\infty}^{\zeta_a(c)} H_a(z) dz$. Assume a mean preserving dispersion of $\mathcal{Z}$. It is also a MPS: $\int_{-\infty}^{a} H(z) dz$ rises, and so $\zeta_B(c) > \zeta_A(c)$.[43] We can rule out $\pi_A(u - \zeta_A(c), c) > \pi_B(u - \zeta_B(c), c)$ for all u. It is because $\pi_a(u - \zeta_a(c), c)$

[43] When $\mathcal{Z}$ has full support, integration by parts requires $\lim_{z \to -\infty} z H(z) < \infty$. By l'Hopital's

is the cdf of $\mathcal{X}+\min\{\mathcal{Z}_a, \zeta_a(c)\}$, we rule out $\mathcal{X}+\min\{\mathcal{Z}_B, \zeta_B(c)\} \succ \mathcal{X}+\min\{\mathcal{Z}_A, \zeta_A(c)\}$ stochastically. This contradicts $E[\mathcal{X}+\min\{\mathcal{Z}_B, \zeta_B(c)\}] = E[\mathcal{X}+\min\{\mathcal{Z}_A, \zeta_A(c)\}]$, as $E[\mathcal{Z}_B] = E[\mathcal{Z}_A]$ and:

$$E[\min\{\mathcal{Z}_a, \zeta_a(c)\}] - E[\mathcal{Z}_a] = \int_{\zeta_n(c)}^{\infty} (\zeta_a(c) - z) dH_a(z) = \int_{\zeta_a(c)}^{\infty} [1 - H_a(z)] dz = c$$

by (3). Altogether, $\pi_B(u - \zeta_B(c), c) - \pi_A(u - \zeta_A(c), c)$ is downcrossing in u. □

rule, $\lim_{z\to-\infty} zH(z) = \lim_{z\to-\infty} -z^2 h(z)$. These limits vanish, for otherwise the second moment $\int_{-\infty}^{\infty} z^2 h(z) dz$ is infinite — impossible, as log-concave densities have finite moments (An, 1997).

E Conditional Recall Chance

In stationary search models, recall never happens and in our two-factor model the recall chance is positive (Lemma 3). One might intuit that the recall chance increases in the gaps between known factors, because the benefits of continuing to search drop more rapidly. Consider two extreme cases: with identical known factors, search is stationary and Sam never recalls before the last stage. But as the gap size explodes, Sam stops immediately, and so never recalls.

For some intuition, proceed conditional on reaching a period. Let r_n be *Sam's recall chance* at stage n, with $1 < n < N$, after he sees all realized known factors, but before he starts searching. Assume no outside option. Sam recalls at stage n if (i) he hits that stage, and (ii) some prior option dominates both it and (iii) dominates the expected value of exploring option $n+1$. So his stage n recall chance is

$$r_n = P(\mathcal{X}_i + \mathcal{Z}_i < \mathcal{X}_n + \zeta \text{ for all } i < n, \mathcal{X}_j + \mathcal{Z}_j > \max\{\mathcal{X}_n + \mathcal{Z}_n, \mathcal{X}_{n+1} + \zeta\} \text{ for some } j < n)$$

Also let s_n be Sam's chance of exploring option n, namely

$$s_n = P(\mathcal{X}_i + \mathcal{Z}_i < \mathcal{X}_n + \zeta \text{ for all } i < n).$$

We make the following claim:

Claim 25 *The conditional recall chance r_n/s_n monotonically rises if all prior gaps $\mathcal{X}_i - \mathcal{X}_{i+1}$, for $i \leq n$, weakly increase.*

Figure 13 illustrates this claim for $n = 2$. The chances r_2 and s_2 are the respective probability measures of the blue shaded area, and the red and blue areas. If the hidden factors $\mathcal{Z}_1$ and $\mathcal{Z}_2$ are uniformly distributed (which is log-concave), then r_n and s_n are proportional to the size of these shaded areas. In this example, an increase in $\mathcal{X}_1 - \mathcal{X}_2$ shifts the blue region left, but does not affect its measure. Since the chance s_2 of reaching stage 2 falls, the ratio r_2/s_2 rises.

We now prove Claim 25. Given the realized known factors, the chance $P(\mathcal{X}_i + \mathcal{Z}_i <$

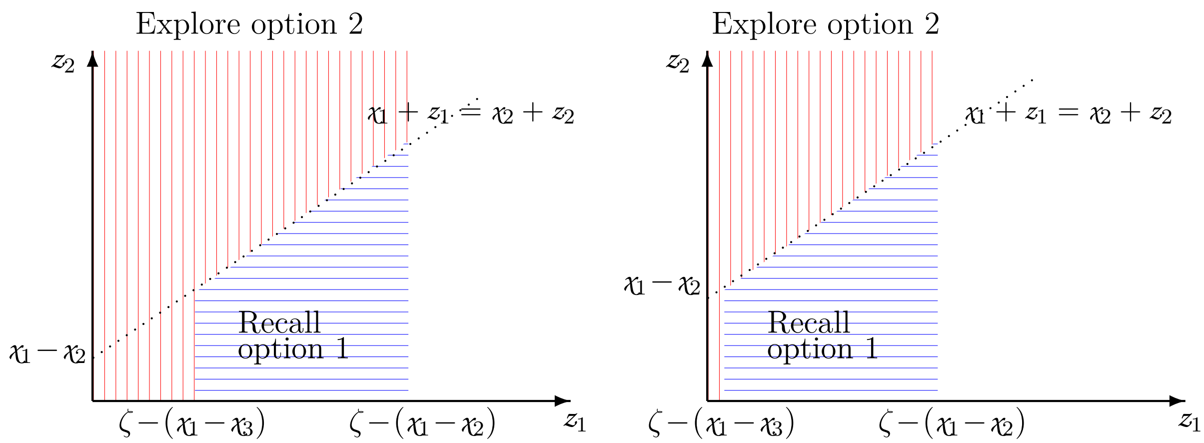

Figure 13: **Conditional recall chance.** The blue shaded area is the event of recalling option 1 at stage 2. Sam hits stage 2 in the red and blue shaded area event. The known factor gap $\varkappa_1 - \varkappa_2$ is larger in the right panel. With a uniform probability measure, the probability r_2 is unchanged while s_2 falls. So the conditional recall chance r_2/s_2 rises.

y for all $i < n$) equals $\Pi_{i=1}^{n-1} H(y - \varkappa_i)$. Hence,

$$r_n = \int_{\varkappa_{n+1}+\zeta}^{\varkappa_n+\zeta} H(y - \varkappa_n) d\Pi_{i=1}^{n-1} H(y - \varkappa_i) = \int_{-(\varkappa_n - \varkappa_{n+1})}^{0} H(\zeta + w) d\Pi_{i=1}^{n-1} H(w + \zeta - (\varkappa_i - \varkappa_n))$$

where the dummy $y = w + (\varkappa_n + \zeta)$ represents the realized value of the best option from 1 to $n-1$. Next, Sam's survival chance can be likewise written as:

$$s_n = P(\varkappa_i + \mathcal{Z}_i < \varkappa_n + \zeta \text{ for all } i < n) = \Pi_{i=1}^{n-1} H(\zeta - (\varkappa_i - \varkappa_n)).$$

Define the cdf J on $(-\infty, 0]$:

$$J(w) \equiv \frac{\Pi_{i=1}^{n-1} H(w + \zeta - (\varkappa_i - \varkappa_n))}{\Pi_{i=1}^{n-1} H(\zeta - (\varkappa_i - \varkappa_n))}$$

Fix $i \leq n$. As H is log-concave, $J(w)$ falls as $\varkappa_i - \varkappa_{i+1}$ rises, fixing other gaps $\varkappa_j - \varkappa_{j+1}$. The conditional recall chance $r_n/s_n = \int_{-(\varkappa_n - \varkappa_{n+1})}^{0} H(\zeta + w) dJ(w)$ rises in $\varkappa_i - \varkappa_{i+1}$. $\square$

F Is Web Search Really Sequential?

Our model exhibits a known property of search and learning models, that encouraging search outcomes need not reduce search duration. Rosenfield and Shapiro (1981) showed that one need not even employ a cut-off strategy — for expectations rise after high draws. When W_1 is larger, so too are $\mathcal{X}_1, \ldots, \mathcal{X}_n$, and expected search duration rises. Our log-concavity assumptions help ensure the optimality of our threshold rule.

So inspired, we econometrically test our model. Intuitively, better earlier outcomes shorten the search process. But this need not be so. Assume that search lasts $T \geq 1$ stages, and the first web site has payoff $W_1 = \mathcal{X}_1 + \mathcal{Z}_1$. Consider the OLS regression $T = \beta_0 + \beta_1 W_1 + \epsilon$ on data generated from our model. We claim that — fixing the CTR $\sigma_1 = 1 - G(u - \zeta(c))^n$ — the true coefficient obeys $\beta_1 > 0$, provided the quit payoff u is large enough and search cost c small enough. By Lemmas 1 and 2, Sam clicks at stage i if $\mathcal{X}_i + \zeta(c) > \Omega_i = \max(u, w_1, w_2, \ldots, w_i)$. In the limit $u \to \infty$ and $c \to 0$, and so $\zeta(c) \to \infty$, the stage i search decision depends only on the known factor, clicking if $\mathcal{X}_i > u - \zeta(c) = \bar{\ell}$. As $W_1 = \mathcal{X}_1 + \mathcal{Z}_1$ is correlated with $\mathcal{X}_2$, one clicks the second web site more often with higher W_1 (Claim 26 (a)) — i.e. $\beta_1 > 0$.

In fact, search duration is not monotone in the first search outcome even ignoring its known factor. For consider the OLS regression $T = \beta_0 + \beta_2 \mathcal{Z}_1 + \epsilon$. The absolute true coefficient $|\beta_2|$ vanishes as $u \to \infty$ and $c \to 0$, fixing the CTR (Claim 26 (b)). This follows once more because the clicking decision depends on $\mathcal{X}_i$ but not $\mathcal{Z}_i$ for large u, very small c, but with $u - \zeta(c)$ fixed. So T and $\mathcal{Z}_1$ are uncorrelated.

Claim 26 *Posit* limit $(\star)$*: the quit payoff u explodes ($u \uparrow \infty$), and the clicking cost vanishes ($c \downarrow 0$) but the CTR holds constant. Then* (a) *the limit coefficient β_1 is positive, and* (b) *the coefficient β_2 tends to* 0*.*

De Los Santos et al. (2012) (DHW) studies an online book market and test three *sine qua non* predictions of sequential search models. In their most relevant "test 3", DHW consider an OLS regression $T = \beta_0 + \beta_3 P_1 + \epsilon$ of the number of searches T on the *price discount* P_1 at the first store. They assume that price discounts are learned after visiting the store,[44] and suggest that Weitzman's model requires $\beta_3 < 0$. For a

[44]DHW posit that consumer i's payoff from buying at store j is $u_{ij} = \delta_{ij} + \alpha_i p_j$. Consumer i

higher first price discount intuitively leads Sam to stop more often. Finding that β_3 is not statistically different from 0, DHW reject Weitzman's model.

But this logic misses selection effects. For a price learned *after* a store visit is best modeled as a hidden factor: $\mathcal{Z}_1 = P_1$. As our second regression shows, Weitzman's model yields a statistically insignificant coefficient β_3 on the hidden factor for a large quit payoff u and search cost c small relative to rewards — a plausible limit in their context.[45] But if Sam learns about the price discount P_1 *before* searching, then $P_1 = \mathcal{X}_1 + \mathcal{Z}_1$, where Sam sees the known factor $\mathcal{X}_1$. In this case, our first regression shows that even $\beta_3 > 0$ is consistent with Weitzman's model when u is large and c is small. So really any sign of β_3 is consistent with Weitzman's model.

While DHW use data for cases *when users purchase from a web site* after searching, our regressions condition on participation. We study the regression $T = \beta_0 + \beta_3 P_1 + \epsilon$ given a final purchase. Venturing the extreme case when $P_1 = \mathcal{X}_1$, we show that if the hidden factor density has a thin tail, then $\beta_3 \geq 0$ as $u \to \infty$ and $c \to 0$, contrary to the DHW conjecture: Higher price discounts do not shorten search.

Claim 27 *If $h(z)$ has a thin tail, then β_3 has a non-negative limit given* $(\star)$.

We also illustrate the insight of Claim 27 numerically. We generate simulated data using our calibrated model (§H) and then run regressions based on it. The parameter values are given by Table 4 and following DHW the fraction of consumers that are aware of one, two, three and four bookstores are 0.35, 0.34, 0.23 and 0.08, respectively. We generate 20,000 web searches and approximately 7700 searches ended with a purchase. The expected number of search is 1.45 and it is 1.54 conditional on purchase. We consider three regression specifications: (1) Regress T on $\mathcal{X}_1$, (2) regress T on $\mathcal{Z}_1$, and (3) regress T on $W_1 \equiv \mathcal{X}_1 + \mathcal{Z}_1$. The results are reported in Table 3. The point is that search duration is positively correlated with the realization of the first known factor and negatively correlated with the first hidden factor. Depending

knows his gross utility δ_{ij} before searching store j, and learns the price discount p_j after visiting store j. In our model, δ_{ij} is the known factor and the price discount p_j is the hidden factor.

[45]For in DHW's data, about 5% of visits to online bookstores result in a transaction (15561 transactions from 327074 searches). Since DHW suggest less than 2 visits per search, the success chance is less than 10%; equivalently, the quitting chance is high, exceeding 90%. This in turn implies that consumers' quit payoffs u must be high relative to the size of the rewards. That consumers search despite such a low success chance implies a small search cost $c > 0$.

on whether price discount is observable before or after search, the correlation between T and P_1 can be of any sign under sequential search.

Table 3: Regression results

Full sample	Specification		
Variable:	(1)	(2)	(3)
First known factor	0.86^{***}		
First hidden factor		-0.21^{***}	
First option payoff			-0.05^{***}
Intercept	-1.14^{***}	1.45^{***}	1.46^{***}
Conditional on purchase			
Variable:	(1)	(2)	(3)
First known factor	0.21^{***}		
First hidden factor		-0.50^{***}	
First option payoff			-0.48^{***}
Intercept	1.45^{***}	1.87^{***}	2.06^{***}
Note:			***p<0.01

PROOF OF CLAIM 26 *(a)*: Let $\mathbb{E}_C$ be the *click-through event* $\mathcal{X}_1 > u - \zeta(c)$ or $T \geq 1$, by (14). The OLS sample estimate of β_1 is $\hat{\beta}_1 = \text{Cov}_e(T, W_1|\mathbb{E}_C)/\text{Var}_e(W_1|\mathbb{E}_C)$, where $\text{Cov}_e(T, W_1|\mathbb{E}_C)$ and $\text{Var}_e(W_1|\mathbb{E}_C)$ are the sample covariance and variance given $\mathbb{E}_C$. Then $\hat{\beta}_1$ converges in probability to $\beta_1 = \text{Cov}(W_1, T|\mathbb{E}_C)/\text{Var}(W_1|\mathbb{E}_C)$ as $N \uparrow \infty$.

Since the cdf of $\mathcal{X}_1$ is $P(\mathcal{X}_1 \leq x_1) = G(x_1)^N$, the conditional expectation

$$E[W_1|\mathbb{E}_C] = \int_{u-\zeta(c)}^{\infty} \int_{-\infty}^{\infty} (x_1 + z_1) dH(z_1) dG(x_1)^N / [1 - G(u - \zeta(c))^N]$$

is constant as $u \to \infty, c \downarrow 0$, fixing $u - \zeta(c) = \bar{\ell}$ (limit $(\star)$). Similarly, since $\text{Var}(W_1|\mathbb{E}_C) = E[W_1^2|\mathbb{E}_C] - E[W_1|\mathbb{E}_C]^2$, the limit variance only depends on $\bar{\ell}$. So the sign of β_1 in this limit depends on $\text{Cov}(W_1, T|\mathbb{E}_C) > 0$. Let $t(x_1, z_1, u, c)$ be the expected number of searches when the user clicks through if $\mathcal{X}_1 = x_1$ and $\mathcal{Z}_1 = z_1$. Then $\text{Cov}(W_1, T|\mathbb{E}_C) = \text{Cov}(W_1, t(\mathcal{X}_1, \mathcal{Z}_1, u, c)|\mathbb{E}_C)$. We derive a formula for $t(x_1, z_1, u, c)$.

Assume $\mathcal{X}_1 = x_1$ and $\mathcal{Z}_1 = z_1$. By Lemmas 1 and 2, the user enters stage n iff

$\mathcal{X}_n + \zeta(c) > \Omega_n = \max(u, w_1, w_2, \ldots, w_n)$. In the limit $u \to \infty$ and $c \to 0$, and so $\zeta(c) \to \infty$, the condition becomes $\mathcal{X}_n > u - \zeta(c) = \bar{\ell}$. By the Markov property of order statistics (footnote 15), the distribution of the known factors of the remaining $N-1$ web sites is the same as $N-1$ *i.i.d.* draws from cdf $G(\chi)/G(\chi_1)$ for $\chi < \chi_1$. So in limit limit $(\star)$, a randomly selected option in the subgame is clicked iff its known factor exceeds $\bar{\ell}$, which occurs with chance $[1 - G(\bar{\ell})/G(\chi_1)]$. Since each of the $N-1$ options is independently clicked with chance $[1 - G(\bar{\ell})/G(\chi_1)]$, the expected number of searches in the limit $u \to \infty, c \to 0$ is $(N-1)[1 - G(\bar{\ell})/G(\chi_1)]$. Fixing $\bar{\ell}$,

$$\lim_{c \to 0, u \to \infty} t(x_1, z_1, u, c) = 1 + (N-1)[1 - G(\bar{\ell})/G(\chi_1)] \equiv \bar{t}(\chi_1). \tag{69}$$

Altogether, $\text{Cov}(W_1, T|\mathbb{E}_C) \to \text{Cov}(\mathcal{X}_1 + \mathcal{Z}_1, \bar{t}(\mathcal{X}_1)|\mathbb{E}_C)$ at the limit. Since $\mathcal{X}_1$ is independent of $\mathcal{Z}_1$ *even* given $\mathbb{E}_C$, $\text{Cov}(\mathcal{X}_1 + \mathcal{Z}_1, \bar{t}(\mathcal{X}_1)|\mathbb{E}_C) = \text{Cov}(\mathcal{X}_1, \bar{t}(\mathcal{X}_1)|\mathbb{E}_C)$. Finally, $\text{Cov}(\mathcal{X}_1, \bar{t}(\mathcal{X}_1)|\mathbb{E}_C) > 0$ as $\bar{t}(\chi_1)$ strictly rises in χ_1 by (69). Altogether, the coefficient $\beta_1 = \text{Cov}(W_1, T|\mathbb{E}_C)/\text{Var}(W_1|\mathbb{E}_C) > 0$ as $u \to \infty$ and $c \to 0$. □

Proof of Claim 26 (b): As N explodes, the OLS estimate $\hat{\beta}_2$ tends in probability to $\text{Cov}(T, \mathcal{Z}_1|\mathbb{E}_C)/\text{Var}(\mathcal{Z}_1|\mathbb{E}_C)$. As $\mathcal{X}$ and $\mathcal{Z}$ factors are independent, the conditional expectation of $\mathcal{Z}_1$ has cdf H under $\mathbb{E}_C$. All told, $\text{Var}(\mathcal{Z}_1|\mathbb{E}_C) = \text{Var}(\mathcal{Z}) > 0$ as $N \to \infty$.

If $(\mathcal{X}_1, \mathcal{Z}_1) = (\chi_1, z_1)$, then the limit $\bar{t}(\mathcal{X}_1)$ of expected search times $t(\chi_1, z_1, u, c)$ as $u \to \infty$ and $c \to 0$ is constant in z_1, by (69). As $t(\chi_1, z_1, u, c) \leq N-1$, the Dominated Convergence Theorem implies that $\text{Cov}(t(\mathcal{X}_1, \mathcal{Z}_1, u, c), \mathcal{Z}_1|\mathbb{E}_C) \to \text{Cov}(\bar{t}(\mathcal{X}_1), \mathcal{Z}_1|\mathbb{E}_C)$. So $\beta_2 \to \text{Cov}(\bar{t}(\mathcal{X}_1), \mathcal{Z}_1|\mathbb{E}_C)/\text{Var}(\mathcal{Z}) = 0$, as $\mathcal{X}_1$ and $\mathcal{Z}_1$ are independent on $\mathbb{E}_C$. □

Proof of Claim 27: Let $\mathbb{E}_P$ be the event that the user eventually purchases. By OLS,

$$\beta_3 = \text{Cov}(T, \mathcal{X}_1|\mathbb{E}_P)/\text{Var}(\mathcal{X}_1|\mathbb{E}_P),$$

where $\text{Cov}(T, \mathcal{X}_1|\mathbb{E}_P)$ and $\text{Var}(\mathcal{X}_1|\mathbb{E}_P)$ are the covariance and variance. Then β_3 is non-negative provided $\text{Cov}(T, \mathcal{X}_1|\mathbb{E}_P) \geq 0$ in the limit $(\star)$.

The user clicks through if he buys, and buys if he clicks through *and* $\mathcal{X}_1 + \mathcal{Z}_1 > u$. So $P(\{\mathcal{X}_1 + \mathcal{Z}_1 > u\} \cap \{\mathbb{E}_P\}) = P(\{\mathcal{X}_1 + \mathcal{Z}_1 > u\} \cap \{\mathbb{E}_C\}) = \int_{\bar{\ell}}^{\infty} [1 - H(u - \chi_1)] dG(\chi_1)^N$.

Since $P(\mathbb{E}_P) = 1 - q = 1 - \pi(u - \zeta(c), c)^N$ by (43), Bayes rule gives:

$$P(\mathcal{X}_1 + \mathcal{Z}_1 > u | \mathbb{E}_P) = \frac{\int_{\bar{\ell}}^{\infty} [1 - H(u - x_1)] dG(x_1)^N}{1 - \pi(\bar{\ell}, c)^N} = \int_0^{\infty} \left[\frac{1 - H(\zeta(c) - s)}{1 - \pi(\bar{\ell}, c)^N} \right] dG(s + \bar{\ell})^N.$$

The limit as $\zeta(c) \to \infty$ as $c \to 0$ of the bracketed term in the integrand is

$$\lim_{\zeta(c)\to\infty} \frac{1 - H(\zeta(c) - s)}{1 - [\int_0^{\infty} g(\bar{\ell} + r) H(\zeta(c) - r)\, ds + G(\bar{\ell})]^N} = \lim_{\zeta(c)\to\infty} \frac{h(\zeta(c) - s)}{N \int_0^{\infty} g(\bar{\ell} + r) h(\zeta(c) - r)\, dr}$$

by l'Hopital's rule, since $\lim_{\zeta(c)\to\infty} [\int_0^{\infty} g(\bar{\ell}+r) H(\zeta(c) - r)\, ds + G(\bar{\ell})] = 1$. In limit $(\star)$:

$$\lim_{c\to 0} P(\mathcal{X}_1 + \mathcal{Z}_1 > u | \mathbb{E}_P) = \lim_{\zeta(c)\to\infty} \frac{\int_0^{\infty} G(s + \bar{\ell})^{N-1} g(s + \bar{\ell}) h(\zeta(c) - s)\, ds}{\int_0^{\infty} g(r + \bar{\ell}) h(\zeta(c) - r)\, dr}. \qquad (70)$$

Write (70) as $\lim_{\zeta(c)\to\infty} E[G(S + \bar{\ell})^{N-1}]$, where the r.v. S has density $g(s + \bar{\ell}) h(\zeta(c) - s)$. Since h has a thin tail, as $\zeta(c) \to \infty$ in the limit $(\star)$, $h(\zeta(c) - s_1) / h(\zeta(c) - s_2) \to 0$ for $s_1 < s_2$ by Claim 11, whence $E[G(S + \bar{\ell})^{N-1}] \to 1$, and so $P(\mathcal{X}_1 + \mathcal{Z}_1 > u | \mathbb{E}_P) \to 1$.

In the limit $(\star)$, since $\lim P(\mathcal{X}_1 + \mathcal{Z}_1 > u | \mathbb{E}_P) = 1$, we have $\mathrm{Cov}(T, \mathcal{X}_1 | \mathbb{E}_P) - \mathrm{Cov}(T, \mathcal{X}_1 | \{\mathcal{X}_1 + \mathcal{Z}_1 > u\} \cap \mathbb{E}_P) \to 0$. Next, $\{\mathcal{X}_1 + \mathcal{Z}_1 > u\} \cap \mathbb{E}_P = \{\mathcal{X}_1 + \mathcal{Z}_1 > u\} \cap \mathbb{E}_C$, as $\mathbb{E}_P \subset \mathbb{E}_C$, while $\{\mathcal{X}_1 + \mathcal{Z}_1 > u\} \cap \mathbb{E}_C$ implies $\{\mathcal{X}_1 + \mathcal{Z}_1 > u\} \cap \mathbb{E}_P$, as the user eventually purchases if he clicks through and the first website dominates the outside option. So $\mathrm{Cov}(T, \mathcal{X}_1 | \{\mathcal{X}_1 + \mathcal{Z}_1 > u\} \cap \mathbb{E}_P) = \mathrm{Cov}(T, \mathcal{X}_1 | \{\mathcal{X}_1 + \mathcal{Z}_1 > u\} \cap \mathbb{E}_C)$, i.e.

$$\lim \mathrm{Cov}(T, \mathcal{X}_1 | \mathbb{E}_P) = \lim \mathrm{Cov}(T, \mathcal{X}_1 | \{\mathcal{X}_1 + \mathcal{Z}_1 > u\} \cap \mathbb{E}_C) \quad \text{in the limit } (\star) \qquad (71)$$

Given $\mathbb{E}_C$, the expected unconditional search time T is the expectation of $t(\mathcal{X}_1, \mathcal{Z}_1, u, c)$, i.e. the mean number of searches when $\mathcal{X}_1 = x_1$, $\mathcal{Z}_1 = z_1$ and the user clicks through:

$$\mathrm{Cov}(T, \mathcal{X}_1 | \{\mathcal{X}_1 + \mathcal{Z}_1 > u\} \cap \mathbb{E}_C) = \mathrm{Cov}(t(\mathcal{X}_1, \mathcal{Z}_1, u, c), \mathcal{X}_1 | \{\mathcal{X}_1 + \mathcal{Z}_1 > u\} \cap \mathbb{E}_C). \quad (72)$$

By equation (69), $t(x_1, z_1, u, c) \to \bar{t}(x_1)$ in limit $(\star)$, which also rises in x. Then $\lim \mathrm{Cov}(t(\mathcal{X}_1, \mathcal{Z}_1, u, c), \mathcal{X}_1 | \{\mathcal{X}_1 + \mathcal{Z}_1 > u\}, \mathbb{E}_C) = \lim \mathrm{Cov}(\bar{t}(\mathcal{X}_1), \mathcal{X}_1 | \{\mathcal{X}_1 + \mathcal{Z}_1 > u\}, \mathbb{E}_C) \geq 0$. So by (71)–(72), $\lim \beta_3 = \lim \mathrm{Cov}(T, \mathcal{X}_1 | \mathbb{E}_P) / \mathrm{Var}(\mathcal{X}_1 | \mathbb{E}_P) \geq 0$ in the limit $(\star)$. $\square$

G Asymptotic Search Duration

Theorem 10 has implications for the estimation of search duration. Suppose the true search model has a two-factor structure and a large number of options, but an econometrician ignores the pre-search information and incorrectly specified a stationary model with a fixed known factor, i.e. $\mathcal{X} = \bar{\chi}$. By Theorem 10, the econometrician will overestimate search duration if G lacks a thin tail. Now we argue that the size of the error can be arbitrarily large. Denote τ^∞ as the search duration in our two factor model as $N \to \infty$. Let τ_s be the expected search duration in a stationary search model, i.e. $\mathcal{X} = \bar{\chi}$ and $N \to \infty$. By Theorem 10, τ^∞ rises in N and converges to τ_s from below as N explodes. We argue that the ratio τ_s/τ^∞ can be arbitrarily large as c vanishes:

Theorem 13 (Asymptotic Search Duration) *Assume G lacks a thin tail. Then τ_s/τ^∞ rises as c falls. As $c \downarrow 0$, τ_s/τ^∞ explodes if and only if H has a thin tail.*

As information technology advances, consumers presumably face more options and smaller search costs. Theorem 13 suggests that, the prediction of a stationary search model can be increasingly misleading as search frictions vanish. We illustrate Theorem 13 with a numerical example in Figure 14.[46] The left panel compares the search duration in our model and that in a stationary model, assuming H has a thin tail. As $c \to 0$, both τ^∞ and τ_s explode, but τ_s increases at a much higher speed. In the right panel, we plot the ratio τ_s/τ^∞ as a function of c. The red line assumes H has a thin tail and the blue line does not. As c vanishes, the blue lines converges to a finite constant while the red line explodes. An implication of this numerical example is that, when conducting counterfactual analysis regarding a reduction of search frictions, the prediction of the model is very sensitive to whether the known factor is degenerate and whether G and H satisfy the thin tail property.

Proof of Theorem 13: In a stationary model, the survival chance of reaching stage n is given by $H(\zeta(c))^n$ and thus the search duration is $\tau_s = 1/[1 - H(\zeta(c))]$.

Next we derive an expression for τ^∞. As $N \to \infty$, Sam will never exercise the

[46] In both panels we assume $\mathcal{X} \sim exp(1)$. The left panel and the red line in the right panel assume $\mathcal{Z} \sim N(0,1)$. The blue line in the right panel assumes $\mathcal{Z} \sim exp(1)$.

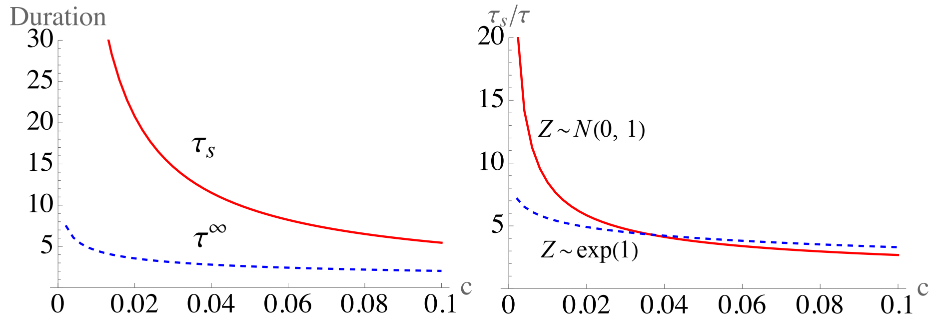

Figure 14: **Comparison of search duration under stationary and nonstationary search. (Theorem 13).** (Left) As c vanishes, the difference between τ_s and τ^∞ rises. (Right) As c vanishes, the ratio τ_s/τ^∞ remains finite when H does not have a thin tail (blue dashed). When H has a thin tail (red), the ratio explodes as c vanishes.

outside option. Therefore, by (7) the survival chance can be rewritten as

$$\sigma_n = E_{\mathcal{X}_n}\left[\left(\frac{\delta(\mathcal{X}_n, c)}{1-G(\mathcal{X}_n)}\right)^n\right] = E_{\mathcal{X}_n}\left[\left(\frac{\int_{\mathcal{X}_n}^\infty H\left(\mathcal{X}_n + \zeta(c) - x\right) g(x)dx}{1-G(\mathcal{X}_n)}\right)^n\right]$$

where the expectation is taken over the realization of the order statistic $\mathcal{X}_n$. As $N \to \infty$, given n, $\mathcal{X}_n \to \infty$ in probability. Hence

$$\begin{aligned}\lim_{N\to\infty} \sigma_n &= \lim_{\chi_n\to\infty}\left(\frac{\int_{\chi_n}^\infty H\left(\chi_n + \zeta(c) - x\right) g(x)dx}{1-G(\chi_n)}\right)^n \\ &= \lim_{\chi_n\to\infty}\left(\frac{\int_0^\infty H\left(\zeta(c) - y\right) g(y+\chi_n)dy}{1-G(\chi_n)}\right)^n = \left(\int_0^\infty H\left(\zeta(c)-y\right) e^{-y\ell}dy\right)^n\end{aligned}$$

where the second line changes variable $y = \chi - \chi_n$ and the last equation uses Claim 10 and 11 and $\ell \equiv \lim_{\chi\to G^{-1}(1)} g(\chi)/[1-G(\chi)]$. Thus, as $N \to \infty$, $\sigma_n = P(\mathcal{Z} < \zeta(c) - Y)^n$ where $Y \sim \exp(\ell)$. Since $\tau^\infty = \Sigma_{n=1}^\infty \sigma_n$,

$$\tau^\infty = \frac{1}{1-P(\mathcal{Z} < \zeta(c) - Y)} = \frac{1}{E[1-H(\zeta(c)-Y)]}. \tag{73}$$

Hence the ratio $\tau_s/\tau^\infty = 1$ if G has a thin tail and $\tau_s/\tau^\infty > 1$ otherwise.

Suppose G does not have thin tail. By $\tau_s = 1/[1-H(\zeta(c))]$ and (73),

$$\frac{\tau_s}{\tau^\infty} = \frac{E[1-H(\zeta(c)-Y)]}{1-H(\zeta(c))}.$$

This ratio falls in c, provided that $1-H$ is log-concave. As c falls to 0, $\zeta(c)$ explodes. By l'Hopital's rule, the limit of the ratio is given by

$$\lim_{c\to 0}\frac{\tau_s}{\tau^\infty}=\lim_{\zeta\to\infty}\frac{E[h(\zeta-Y)]}{h(\zeta)}.$$

If H has a thin tail, then this limit explodes, otherwise it is a positive constant. □

Consider the limit when the number of options is large, i.e. $N\to\infty$. We ask what happens when search frictions is vanishing small.

Proposition 2 *If G has a thin tail and H lacks a thin tail, then τc converges to a positive limit as $c\to 0$, otherwise $\tau c\to 0$.*

PROOF: Suppose the distribution G has thin tail, then $\tau\to 1/[1-H(\zeta(c))]$ as $N\to\infty$. The total expected search cost is τc. Consider the limit as search friction vanishes $c\to 0$. At the limit $\tau\to\infty$ because $\zeta(c)\to\infty$. Apply the L'Hospital rule to derive the limit of τc:

$$\lim_{c\to 0}(\tau c)=\lim_{c\to 0}\frac{c}{1-H(\zeta(c))}=\lim_{c\to 0}\frac{1-H(\zeta(c))}{h(\zeta(c))}.$$

The right side is positive and it vanishes at the limit if and only if H has a thin tail.

If G lacks a thin tail, then by (73),

$$\lim_{c\to 0}(\tau c)=\lim_{c\to 0}\frac{c}{E[1-H(\zeta(c)-Y)]}=\lim_{c\to 0}\frac{1-H(\zeta(c))}{E[h(\zeta(c)-Y)]}.$$

If H is log-concave, then the right side vanishes as $\zeta\to\infty$. □

These results are sensitive to the order at which the limits are taken. If we take $c\to 0$ first, then the search duration is always $\tau=N$ and the total search cost is $Nc\to 0$, for any choice of N.

H Calibration of the Example in Figure 6

We now explain the example in Figure 6. We assume Gaussian known and hidden factors $\mathcal{X} \sim N(\mu_x, \sigma_x^2)$ and $\mathcal{Z} \sim N(\mu_z, \sigma_z^2)$. We normalize $\mu_z = 0$. We normalize σ_z to 1 by scaling c, u and $\mathcal{X}$ and normalize μ_x to 0 by subtracting its value from u.

The remaining parameters to calibrate are (u, c, σ_x, N). We calibrate the model to match the purchase chance, search duration, recall chance and the size of the consideration set in the online book market studied by De Los Santos et al. (2012). Their dataset is provided by ComScore and includes detailed online browsing and transaction data from various Internet users. Approximately, 38 percent of the users in their sample realized a transaction in 2002, which leads to transactions from 15 online bookstores with 7,558 observations. Among the transactions in which a consumer visited more than one store, 58 percent exhausted all bookstores in their consideration set and 38 percent recalled a previously visited store. In the sample, the fraction of consumers that are aware of one, two, three and four bookstores are 0.35, 0.34, 0.23 and 0.08, respectively. When calibrating the parameters, we set $N = 1, 2, 3, 4$ and compute the average statistics using the same weights.

We use a Monte Carlo method to compute the average statistics when calibrating the model. For each set of parameter values, we simulate 1000000 times to compute the relevant average statistics. Then we look for the value of (u, c, σ_x) such that the mean-square difference between the average statistics and the data is minimized. We report the calibrated parameters in the following table:

Parameter	Description	Target	Value
u	Quit payoff	Fraction of users realized a product transaction	0.78
c	Search cost	Fraction of transactions that exhaust all options given that two or more stores are explored	0.06
σ_x	Standard deviation of $\mathcal{X}$	Fraction of transactions that end with recall given that two or more stores are explored	0.44

Table 4: Parameter values of the calibrated model

Comparing search with and without pre-search information we assume $u = 0.78$,

$c = 0.06$, $\mathcal{X} \sim N(0, 0.44^2)$, $\mathcal{Z} \sim N(0, 1)$ and $N = 5$ when there is information. When there is no pre-search information, we assume $\mathcal{X} \sim N(0, 0)$ and $\mathcal{Z} \sim N(0, \sqrt{1 + 0.44^2})$ so that the distribution of the sum $\mathcal{X} + \mathcal{Z}$ is unchanged. For each set of parameter values, we use the formula in (7) to compute search duration in our model.

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